

# **ATC 63 Methodology for Evaluating Seismic Collapse Safety of Archetype Buildings**

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## **OVERVIEW**

This is one of four companion papers describing the Applied Technology Council project (ATC-63) to develop a methodology to assess seismic design provisions for building systems. This paper describes the underlying approach to evaluate the collapse safety of a set of archetype buildings, whose designs reflect the key features of the seismic design requirements. The companion papers provide a broader overview of the ATC 63 project [Kircher and Heintz, 2008] and two application studies to reinforced concrete moment frames [Haselton et al. 2008] and wood-frame buildings [Filiatrault et al. 2008]. Following an overview of the assessment methodology, this paper reviews specific aspects related to (a) modeling collapse assessment of buildings by nonlinear time-history analysis, (b) development of collapse fragility curves, including variability due to design, construction, and modeling uncertainties, (c) ground motion characteristics with adjustment for spectral shape effects for collapse assessment, (d) evaluation and acceptance criteria for archetype building models.

## **COLLAPSE SAFETY ASSESSMENT METHODOLOGY**

The overall ATC-63 assessment process is illustrated in Figure 1. It assumes that one begins with a well defined concept of the proposed seismic force-resisting system, including the type of construction materials, system configuration, inelastic dissipation mechanisms, and intended applications (building layouts, heights, etc.). The formal assessment process begins with the establishment of a clearly articulated set of design provisions, substantiated by testing and analyses to characterize the inelastic response of the structural system components. The design requirements typically include the general requirements of the type in ASCE 7 [2005], e.g., seismic response factors ( $R$ ,  $C_d$ ,  $\Omega_o$ ), drift limits, height and usage restrictions (if any), and specific design and detailing requirements from appropriate material standards.

The next step is to develop a series of archetype models of the structural system, which reflect the range of applications and seismic behavioral aspects of the system. This requires defining an idealized model that reflects salient design features that affect the collapse response of the structural system. For example, for ductile moment resisting frames, the three-bay multi-story frame model, shown in Figure 2, is considered sufficient to capture the important behavioral effects in beams, columns and beam-column joints that govern collapse behavior. The three-bay configuration is judged to be the minimum necessary to capture overturning forces in columns and a mix of interior and exterior columns and joints. Multiple realizations of the idealized archetype models are then designed to represent the expected range of building heights, bay widths, gravity load ratios, and Seismic Design Categories for the archetype system model.

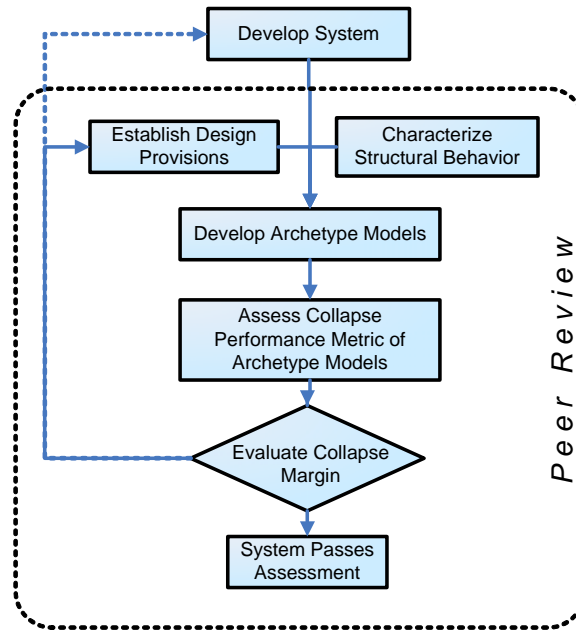


FIGURE 1 – SCHEMATIC FLOWCHART OF ATC-63 METHODOLOGY FOR SYSTEM PERFORMANCE ASSESSMENT

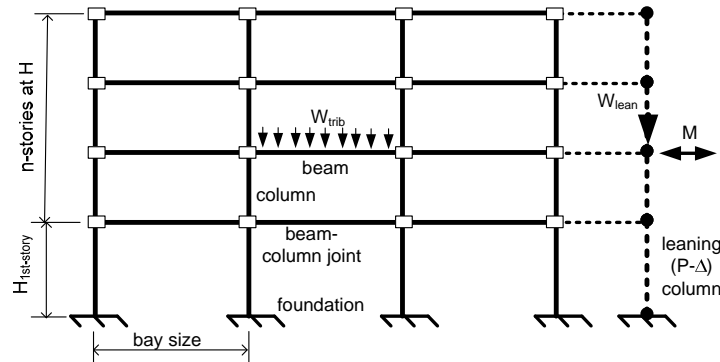


FIGURE 2 – IDEALIZED ANALYSIS MODEL FOR SPECIAL MOMENT FRAME ARCHETYPE

The collapse capacity of archetype model is evaluated by nonlinear time-history analyses for a prescribed set of ground motions whose amplitudes are scaled to reflect specified earthquake ground shaking intensities. The nonlinear assessment must account for all likely modes of strength and stiffness degradation that can lead to earthquake-induced collapse. Ideally, all of the likely modes of failure are incorporated directly in the time history simulation; however, the method makes provision to check collapse modes that are not simulated directly in the analysis.

The collapse capacities of the archetype models are then evaluated to determine whether the proposed system and design requirements provide adequate collapse safety. The acceptance criterion is expressed as a minimum required collapse margin between the median value of the collapse capacity and the intensity of the Maximum Considered Earthquake (MCE) for which the archetype models are designed. The assessment incorporates statistics from the nonlinear analysis and other factors that account for uncertainties in the nonlinear structural component and system behavior and how accurately the archetype models represent actual conditions. As

shown by the return arrow in Figure 1, where the system does not meet the required performance, one can iterate on the system definition and/or design provisions to increase the collapse capacity. For example, one could revise the seismic response factor, i.e., R-value, to adjust the design force requirements, or alternatively, revisions could be made to other parameters (such as capacity design or detailing requirements) to adjust the strength and deformation capacity of the structure.

## NONLINEAR TIME HISTORY MODELING OF COLLAPSE

The collapse assessment is done using nonlinear time history analysis, a key aspect of which are models that capture strength and stiffness degradation at large deformations. Collapse modes can generally be distinguished between sidesway and vertical collapse. Sidesway collapse occurs when the lateral strength and stiffness become insufficient to resist destabilizing P- $\Delta$  effects, resulting in large interstory drifts. Vertical collapse can arise due to loss in vertical load carrying capacity of one or more components in the structure, such as punching failure at a slab-column joint or loss in axial capacity of a column. Assessment of sidesway collapse of indeterminate systems is best accomplished through nonlinear response analysis of the entire system to account for inelastic force redistribution. On the other hand, vertical collapse, which is generally more difficult to simulate directly, tends to be more localized and can be evaluated on a component-wise basis using imposed deformations and forces from the overall analysis.

Using as an example reinforced-concrete moment frames, accurate modeling of strength and stiffness degradation leading to sidesway collapse can be achieved by integrating degrading hinge-type models of the type shown in Figure 3 into the archetype analysis models of Figure 2. A few key aspects of this model are the characterization of the post-peak softening branch of the monotonic backbone curve (Figure 3a) and the degrading hysteretic response (Figure 3b). As shown by Ibarra et al. [2005], the post-cap degrading portion of the monotonic backbone curve is essential to simulating collapse due to the combined effects of inelastic softening and P- $\Delta$  effects. For the reinforced concrete frame examples described in the companion paper [Haselton et al. 2008], Haselton and Deierlein [2007] calibrated the concentrated spring model of Figure 3

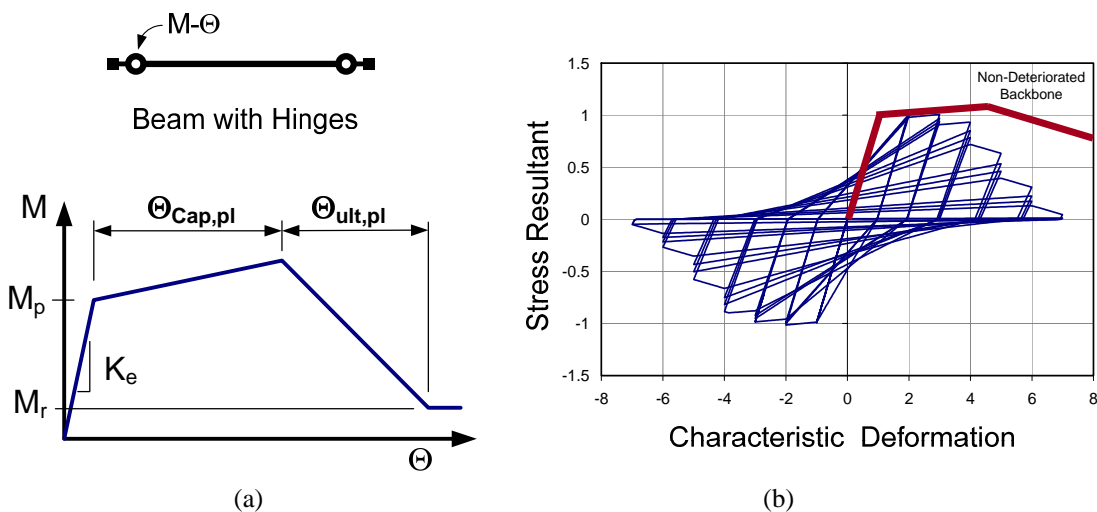


FIGURE 3 – EXAMPLE OF CONCENTRATED HINGE-TYPE MODEL FOR NONLINEAR FRAME ANALYSIS  
(A) MONOTONIC BACKBONE CURVE, AND (B) HYSTERETIC RESPONSE

to simulate nonlinear response of reinforced concrete beams, columns, and beam-column joints. They calibrated the model parameters to the mean values of component behavior, so as to be consistent with the premise of simulating the expected value of the collapse response. Uncertainties in the model parameters and their effect on the collapse assessment are incorporated later through adjustments to the collapse fragility.

## INPUT GROUND MOTIONS

Since the ATC-63 method is geared toward assessing building code design provisions that can be generally applied to any geologic site, the input ground motions and hazard information are developed in a generic sense. The ATC-63 guidelines include two suites of ground motions and procedures for scaling these relative to the seismic hazard intensities of the Seismic Design Categories. One set of records, termed the “Far-Field” record set, includes twenty-two ground motion pairs recorded at sites located greater than 10 km from fault rupture. The second “Near-Field” set includes twenty-eight pairs of motions recorded at sites located within 10km of the fault. Records in each set were selected to provide an unbiased suite of motions that represent strong ground motion shaking with earthquake magnitudes of 6.5 to 7.9. Within each set, the records are normalized by their peak ground velocities to reduce the scatter while preserving variations that are consistent with variations observed in ground motion attenuation functions.

To assess the collapse capacity by nonlinear time history analysis, the amplitude of the ground motion records are scaled based on the fundamental vibration period of the building under consideration. Shown in Figure 4a are the spectra of the normalized Far-Field record set, including the statistical average spectrum. The four MCE demand spectra, shown in Figure 4b, reflect the upper and lower bound hazard levels associated with Seismic Design Categories B through D of ASCE 7 (2005). Superimposed on the demand spectra are the scaled average spectra of the Far-Field record set, where the spectral values are matched at a period of 1 second. When used to perform nonlinear analyses of specific archetype analysis modes, the ground motion set is scaled based on the fundamental period of each specific model.

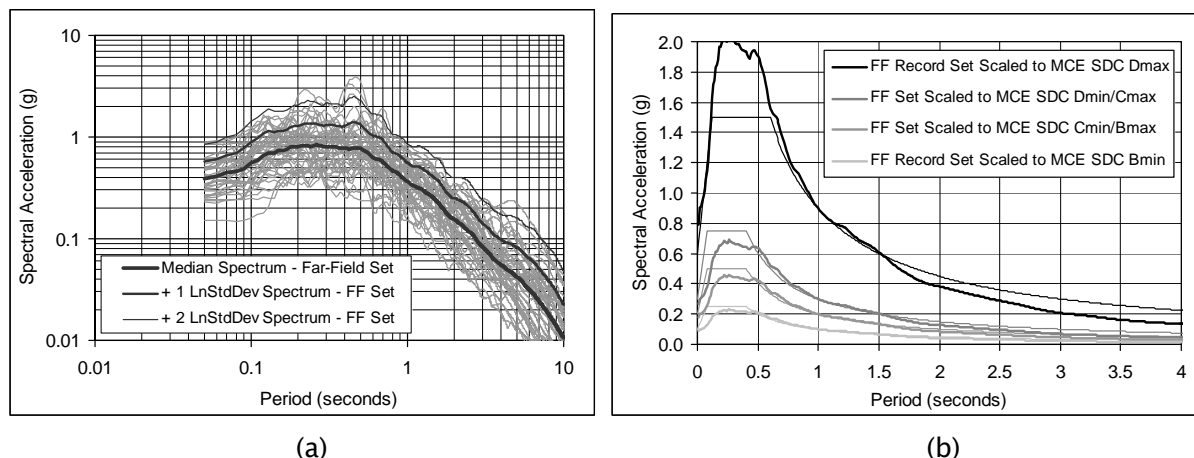


FIGURE 4 – INPUT GROUND MOTIONS (A) RESPONSE SPECTRA PLOTS, (B) EXAMPLE OF INTENSITY ANCHORING TO MAXIMUM AND MINIMUM MCE DESIGN SPECTRA FOR SDC B, C AND D

## INCREMENTAL DYNAMIC ANALYSIS

Incremental Dynamic Analysis (IDA) is a technique to systematically process the effects of increasing earthquake ground motion intensity on structural response up to collapse [Vamvatsikos and Cornell, 2002]. Shown in Figure 5 are results of IDA for a four-story reinforced concrete special moment frame that was analyzed for each of the 44 ground motion records of the Far-Field record set at increasing intensity. Each point in the figure represents the results of one nonlinear time-history analysis – relating the spectral intensity of the record to the peak interstory drift recorded during the analysis. Each curve represents the response of the structure to a single ground motion whose intensity is increased until collapse is detected when excessive drifts occur under small increases in ground motion intensity. Where non-simulate vertical collapse modes are detected, these would have the effect to cut off the IDA curves before they reach the simulated sidesway collapse modes.

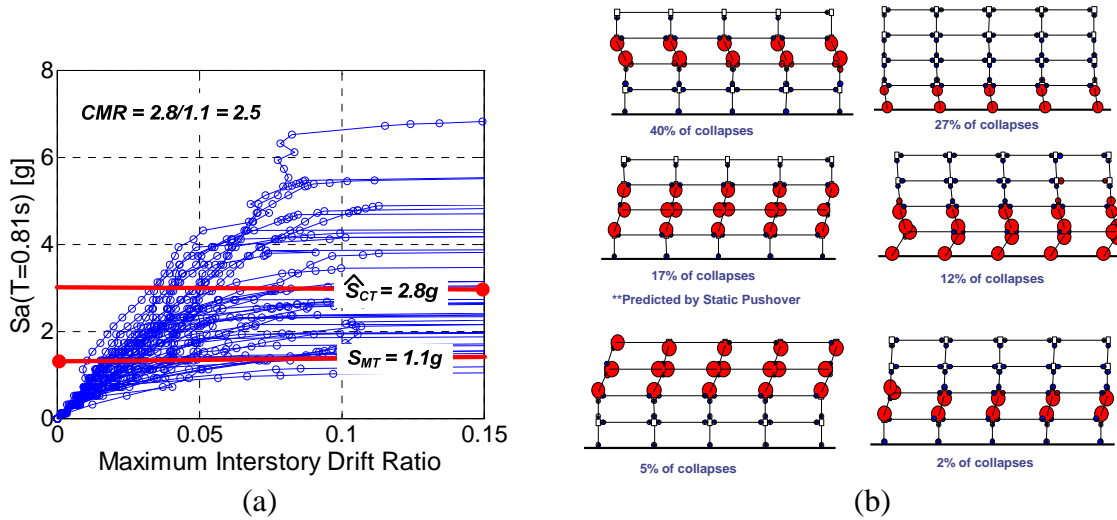


FIGURE 5 – ILLUSTRATION OF INCREMENTAL DYNAMIC ANALYSIS (IDA) RESULTS FOR A 4-STORY FRAME (A) IDA PLOT OF GROUND MOTION INTENSITY VERSUS BUILDING DRIFT, (B) BUILDING COLLAPSE MODES

The ATC-63 methodology adopts a strategy of scaling the ground motions on a set-wise basis, where the intensity measure for all of the records in the set is based on the median response spectra of the entire record set. Further, the scaling of the spectral average is done at the period of the first (or natural) mode of vibration of the structure. Back in Figure 4a, the spectra were scaled based at a period of 1 seconds, but for each specific analysis the spectral intensity measure is based on the fundamental period for each analysis model. For example, the results shown in Figure 5a were scaled at a period of 0.8 seconds, which is the natural period of the four-story reinforced concrete frame model in this example. By scaling the entire record set with a consistent scale factor, the record set could be considered as a ground motion earthquake scenario whose intensity is based on the average of the set.

The large variability in the IDA response plots (Figure 5a) reflects both the spectral variability of each record about the median (see Figure 4a) and other features of the ground motions, e.g., duration and frequency content, which are not fully reflected in the spectral acceleration intensity. The inherent variability that different records have on the structural response is further evident in the variety of collapse modes observed in the four-story frame. As shown in Figure 5b, even though the frame in this example conformed to modern capacity design standards,

collapse occurred due to a single story mechanism in over two-thirds of the records (40% caused a third story mechanism, 27% a first story mechanism, and 2% a second story mechanism).

Despite occurrence of story mechanisms, the results in Figure 5a indicated that the four-story frame performed well in that the calculated collapse intensities generally exceeded the MCE intensity by a large margin. Referring to Figure 5a, the median collapse capacity of  $S_{CT} = 2.8g$  is about 2.5 times the MCE intensity of  $S_{MT} = 1.1g$ . These IDA collapse statistics are re-plotted in the collapse fragility curve of Figure 6a, which relates the probability of collapse to the spectral intensity of the ground motion. Characterized by a lognormal distribution, the collapse fragility curve is a cumulative distribution function, defined by the median collapse intensity ( $S_{CT} = 2.8g$ ) and the dispersion or standard deviation of the natural log,  $\sigma(\ln(S_a)) = 0.45$ , both of which are obtained from the IDA data. Reading off the graph at the MCE intensity of  $S_a = 1.1g$ , this collapse fragility indicates that the probability of collapse at the MCE intensity is less than 2% (i.e.,  $P_{collapse}[S_a = S_{MT}] < 0.02$ ). However, the plot of Figure 6a is an interim collapse fragility curve that does not account for modeling uncertainties and other aspects of the ground motions that are important for accurate characterization of collapse. These are considered next.

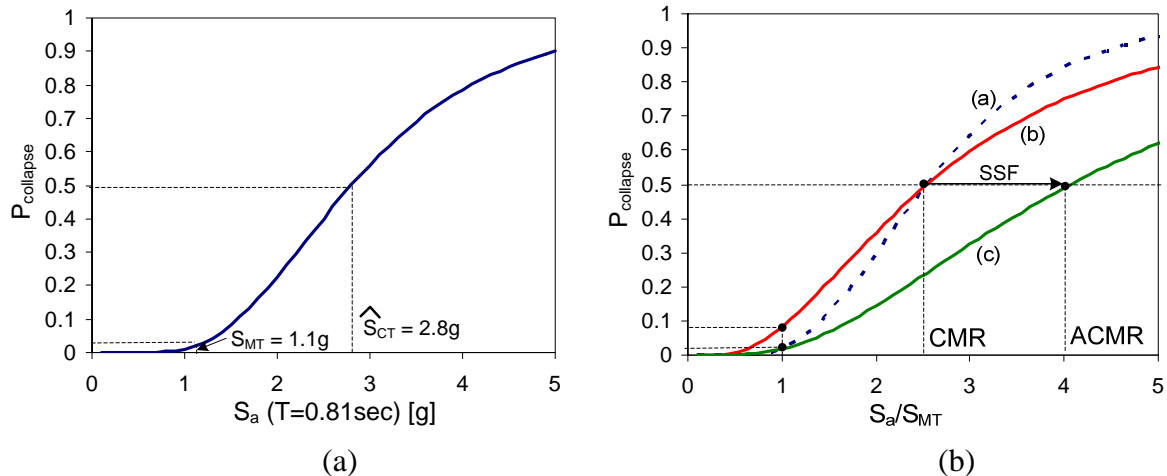


FIGURE 6 – COLLAPSE FRAGILITY CURVE (A) BASIC CURVE FROM IDA ANALYSIS, (B) MODIFICATIONS TO ACCOUNT FOR MODELING UNCERTAINTY AND SPECTRAL SHAPE EFFECTS

## COLLAPSE FRAGILITY CURVE

Two adjustments need to be applied to the IDA collapse fragility before the collapse assessment is considered complete. The first modification is to adjust for modeling uncertainties, and the second is to adjust for the unique spectral shape effects of extreme (rare) ground motions that cause collapse.

**Modeling Uncertainty:** As described until now, the nonlinear analysis model has been based on the mean (expected or average) properties of the structure, such that the only uncertainties reflected in the collapse assessment are those associated with the variations in response for alternative ground motions. Studies by Ibarra et al. [2005] and Haselton and Deierlein [2007] have shown that certain model parameters, such as the inelastic capping rotation and post capping slope (see Figure 3a) can have a significant effect on the collapse performance. One way to account for variability in response introduced by uncertainties in the structural model is by adjusting the dispersion ( $\sigma(\ln(S_a))$ ) of the collapse fragility curve. The increased dispersion

is evident by comparing curves (a) and (b) in Figure 6b, where the dashed curve (a) is the same as the basic IDA fragility curve of Figure 6a, reflecting uncertainties due to the nonlinear response to ground motions, and the solid curve (b) includes the additional modeling uncertainty.

From detailed studies of nonlinear reinforced concrete component response and its effect on the four story frame example, Haselton and Deierlein [2007] have shown the modeling uncertainty to increase the dispersion in the collapse fragility curve from  $\sigma(\ln(Sa)) = 0.45$  (for record uncertainties) to  $\sigma(\ln(Sa)) = 0.65$  (for combined record and modeling uncertainties). This reflects, for example, variability in the capping point rotation (Figure 3a) of  $\sigma(\ln(\theta_{cap})) = 0.6$ , which is carried through the nonlinear time history and IDA analyses along with other factors to result in the cumulative collapse intensity uncertainty of  $\sigma(\ln(Sa)) = 0.65$ .

The horizontal axis in Figure 6b has been normalized by the MCE intensity. Thus, the revised horizontal index represents the ratio between the earthquake intensity that causes collapse and the MCE intensity for which the structure has been designed. The collapse ratio at the median point is termed the *Collapse Margin Ratio* (CMR). The  $CMR = 2.5$  for the median in Figure 6b is the same median margin as in Figure 5a and is unchanged by adjustments due to the uncertainty. While the median margin is unchanged, the modeling uncertainties increase the probability of collapse at the MCE (at  $S_a/S_{MT} = 1$ ) from less than 2% (Figure 6) to about 8%.

**Spectral Shape Effect:** Haselton and Baker [2006] have demonstrated the importance of considering the unique spectral shape of extreme ground motions when evaluating collapse. Since code-conforming structures are expected to resist ground motions on the order of one to three times the MCE intensity, and since the MCE typically has a return period of 1000 to 2500 years, then the median collapse intensities are very infrequent.

Generally speaking, high intensity MCE ground motions in coastal California (and similar regions) are *infrequent* ground motions that can occur under rather *frequent* earthquakes. For example, shown in Figure 7 are the hazard spectra calculated from the Boore/Joyner/Fumar (BJF) attenuation function for a magnitude 6.9 earthquake at a distance of 11 km. Return periods of such earthquakes on active faults are on the order of 150 to 500 years. The thick line corresponds to the median value from the BJF attenuation functions, and each of the lines above and below represent extreme values, corresponding to  $\pm 1\varepsilon$  and  $\pm 2\varepsilon$ , where  $\varepsilon$  is a standard measure of the variability in ground motions. Superimposed on this plot is a response spectra for a ground motion recorded motion from the Loma Prieta earthquake, which is consistent with the M6.9 and 11 km distance used in the attenuation estimates. At a period of  $T=1$  second, the Loma Prieta record has a spectral acceleration of about  $0.9g$ , which is  $+2\varepsilon$  above the BJF median and which is representative of the MCE intensity for a Seismic Design Category D design.

The Loma Prieta record, shown in Figure 7, can be referred to as a “ $+2\varepsilon$  record at  $T=1$  seconds”. Extreme spectral values like these govern the high intensity long return period hazard values. An important feature of such motions is that they generally do not have high spectral values at all periods, but instead tend to drop off at periods away from the index period. For example, at  $T=0.45$  seconds this Loma Prieta motion has a neutral “zero  $\varepsilon$ ” intensity and at  $T=2$  seconds has a  $+1\varepsilon$  intensity. Thus, the  $\varepsilon$  parameter is a function of both the ground motion and the period at



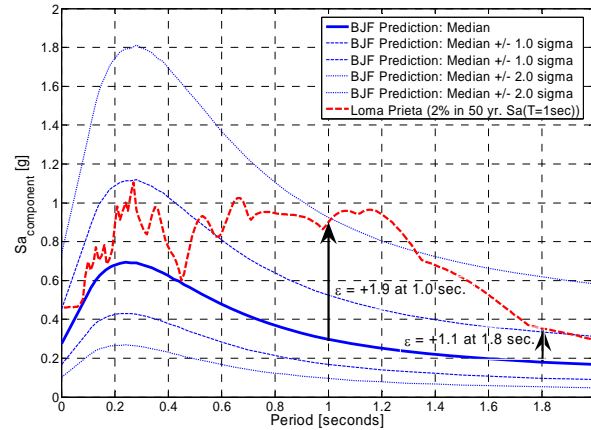


FIGURE 7 - COMPARISON OF AN OBSERVED GROUND MOTION SPECTRUM WITH SPECTRA PREDICTED BY AN ATTENUATION FUNCTION (HASLTON AND BAKER 2006)

which the spectral quantity is evaluated. This spectral shape characteristic is important, since records whose intensities that drop off at higher periods will tend to be less damaging than typical (lower intensity) records whose intensities do not drop off.

When scaling ground motions to represent extreme (rare) shaking intensities for a certain period range (typically near the fundamental vibration mode), this “spectral shape” factor can have a large effect on collapse. In nonlinear IDA simulations, this effect can be included by either (a) choosing ground motions that have positive  $\varepsilon$  values at the predominate period of the structure, or (b) adjusting the collapse fragility to account for the spectral shape effect. The ATC-63 methodology follows the latter approach since it can be implemented with a single set of ground motions that can be used and scaled at different periods for different archetype model systems.

For buildings qualified for use in Seismic Design Category D, where the MCE hazard tends to be dominated by positive  $\varepsilon$  records, the ATC-63 method specifies a shift of up to 1.65 times in the median collapse intensity to account for spectral shape. Aside from Seismic Design Category (which reflects the seismological characteristics), the median shift is a function of the archetype system period and its ductility. The shift is largest for ductile systems (as judged by a pushover analysis) with long periods (greater than 1 to 1.5 seconds). Referring back to the example of Figure 6b, for this ductile four-story reinforced concrete frame system, the spectral shape factor, SSF, is 1.6. Referring to Figure 6b, the CMR is multiplied by the SSF to shift the fragility curve to the right. The new median point, called the Adjusted Collapse Margin Ratio (ACMR) anchors the shifted fragility curve. The result is a significant reduction in the probability of collapse at the MCE – on the order of a 4X reduction in this case.

## RELATING ACMR TO COLLAPSE PROBABILITIES

As illustrated in Figure 6b, the adjusted collapse margin ratio, ACMR, is related to the probability of collapse at the MCE ground motion intensity (ACMR = 1.0) by the assumed lognormal form of the collapse fragility relationship and the variability (dispersion) in the collapse assessment. The ATC 63 procedure provides guidelines on assessing the composite uncertainties as a function of (i) record-to-record uncertainty in the ground motions, (ii) quality of the nonlinear analysis model, (iii) quality of the available test data to calibrate the nonlinear



analysis and component limit states, and (iv) comprehensiveness and quality of the seismic system design requirements. The composite uncertainties range from a low value of  $\sigma(\ln) = 0.55$  (superior systems and data) to a high value of  $\sigma(\ln) = 1.15$  (for less well-defined systems and data). The low value of 0.55 is largely controlled by the inherent variability in the nonlinear response of structures to randomness in the earthquake ground motions. Added to this are uncertainties associated with the quality and certainty of design/construction provisions and our confidence in predicting the structural behavior and simulating collapse.

Shown in Figure 8 are plots of how the probability of collapse at the MCE relates to the ACMR as a function of the dispersion. Based on benchmark studies of buildings designed per current building codes, the ATC 63 method is targeting maximum permitted MCE collapse probabilities of 10% to 20% (or, conversely, 90% and 80% probabilities of non-collapse). For systems where the design provisions and nonlinear simulation models are well established, the dispersion is  $\sigma(\ln) = 0.65$ . Accordingly, the minimum permissible ACMRs are 2.3 and 1.73, respectively, to meet the MCE collapse probabilities of 10% and 20%. In other words, the *median* collapse probabilities (as determined by the nonlinear analysis and adjusted using the SSF factor) need to be at least 2.3 and 1.73 times the intensity of the MCE hazard. Reducing the uncertainty to  $\sigma(\ln) = 0.55$  reduces the minimum required ACMRs to 2.0 and 1.6, whereas a larger uncertainty of  $\sigma(\ln) = 1.15$  increases the minimum required margins to ACMRs of 4.3 and 2.6.

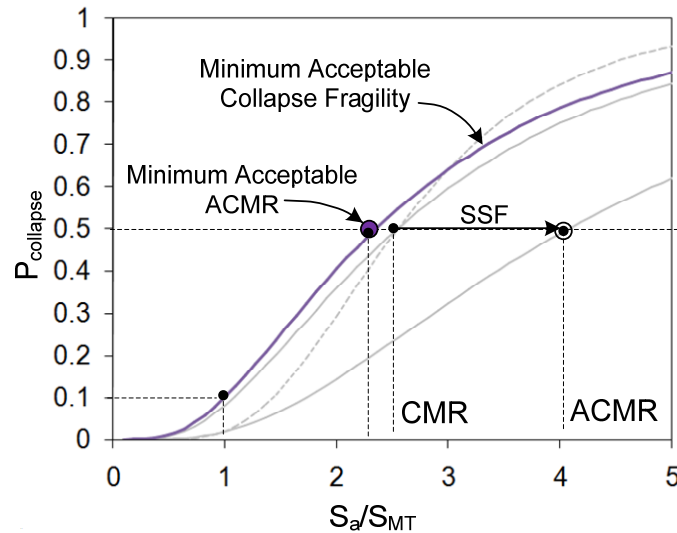


FIGURE 8 – EVALUATION OF CALCULATED CMR AND ACMR AGAINST THE MINIMUM ACCEPTABLE ACMR CRITERIA

The minimum acceptance criteria is described in terms of an *acceptable ACMR*, against which the calculated ACMR for individual structure archetype models or bins of models are compared. The acceptable ACMR is determined as a function of the composite uncertainty for the system in question and a limit on the archetype collapse risk at the MCE intensity. As illustrated in Figure 8, the key calculations required by the methodology are as follows:

- Calculate CMR, ductility, SSF, and ACMR using the median collapse point for each archetype analysis model.
- Determine the composite uncertainty for the structural system under consideration along with *acceptable ACMRs* for individual archetype models and sets of models. Each set of

archetype models should reflect a range of building heights and other key parameters that are expected to influence the archetype model collapse fragility.

- Compare the minimum acceptable ACMR to the calculated ACMRs for individual model structures and for bins of index models. Thus, the required calculations emphasize the determination of mean collapse capacities, rather than the full IDA curve.

The ATC 63 methodology employs performance-based concepts that provide a more consistent and scientifically-based method to assess the collapse safety of buildings. The approach is enabled by research to improve understanding of ground motions and their effects on structural response, nonlinear behavior and computer response simulation of structures, and practical probabilistic approaches to account for the inherent uncertainties in design and analysis. While the immediate focus of the ATC 63 project is on assessing building system response factors for seismic building code provisions, the method and concepts can readily be applied more broadly for the performance assessment and design.

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