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SIMULATING STRUCTURAL COLLAPSE DUE TO EARTHQUAKES: MODEL IDEALIZATION, MODEL CALIBRATION, AND NUMERICAL SOLUTION ALGORITHMS

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Abstract. *This paper discusses the key aspects of simulating structural collapse due to earthquakes using nonlinear dynamic analysis. Emphasis is on specific modeling assumptions associated with the significant strength and stiffness degradation up to the onset of collapse. Various approaches to calibrate models to experimental data are examined to illustrate the importance of accurately distinguishing between in-cycle versus cyclic strength and stiffness deterioration. A key challenge in this regard is that existing experimental data is often insufficient to uniquely calibrate nonlinear models, such that considerable judgment must be exercised in the model calibration. The paper also discusses numerical solution algorithms for obtaining robust, reasonable simulations of collapse, and techniques for avoiding numerical problems associated with a sparse mass matrix. The paper closes with notes on some of the future research needs to simulating structural collapse.*

1 INTRODUCTION

Analytical capabilities for predicting seismic collapse of structures have improved significantly in recent years, through the development of faster and more powerful computers, accurate nonlinear structural component models to simulate strength and stiffness degradation, and robust numerical solution algorithms.

Simulation of structural collapse is increasingly used to evaluate the seismic safety of building structures, including both applications to the assessment and design of specific structures and, more generally, to assess changes to building code provisions. For example, the ATC 58 draft *Guidelines for Seismic Performance Assessment of Buildings* [ATC 2009] including provisions to assess the collapse performance of new and existing buildings. And, the recently published FEMA P695 report on *Quantification of Building Seismic Performance Factors* establishes a procedure whereby analytical assessment of seismic collapse risk is used to evaluate the adequacy of seismic code provisions of new structural systems [FEMA 2009]. These and other initiatives illustrate the inroads of collapse assessment techniques into engineering practice and the associated need for engineers to become well-versed in methods of collapse prediction and the associated challenges with simulating structural behavior up to the point of collapse.

The purpose of this paper is to identify and discuss the important issues involved in simulating earthquake-induced structural collapse. The identification of key difficulties in simulation of structural collapse and proposed solutions are based on the authors' experience over the past several years with modeling structural collapse of reinforced concrete (RC) buildings [Haselton and Deierlein, 2007; Liel and Deierlein 2008]. The paper is envisioned to help frame the discussion of modeling issues that are important for collapse assessment.

Simulation of earthquake-induced structural collapse using nonlinear dynamic analysis is an important *single component* of *broader assessment* of seismic collapse performance, which seeks to evaluate the risk of seismic collapse associated with a particular structure or group of structures. This paper focuses on this topic of directly simulating structural collapse. The overall seismic collapse performance assessment would also include other issues which are outside of the scope of this paper, such as ground motion selection and scaling, and treatment of uncertainties (structural modeling uncertainties, etc.). Some of these other topics are discussed in recent publications by the authors [Haselton et al. 2009, Haselton and Deierlein 2007, Liel et al. 2009].

2 STEPS FOR ASSESSMENT OF SEISMIC COLLAPSE PERFORMANCE

Generally speaking, the steps involved in collapse performance assessment are as follows:

- Define **Goals** of Collapse Performance Assessment *(Section 3)*: What is the purpose of the collapse assessment and what metrics will be used to define collapse risk?
- Develop **Structural Model** Capable of Simulating Collapse *(Section 4)*: Understand damage and failure modes possible in structural system; develop simulation model for components and structural system capable of capturing critical collapse modes; calibrate component models to experimental data, focusing on parameters important for predicting collapse
- Clearly **Define Collapse** limit state *(Section 5)*
- Perform **Nonlinear Dynamic Collapse Analysis** *(Section 6)*: Select and scale set of input ground motions for analysis (outside scope of paper); develop and apply robust numerical solution algorithm; conduct incremental dynamic analysis to establish col-

lapse fragility curve that relates the likelihood of collapse to the ground motion intensity; check the overall accuracy of the collapse assessment against expected performance.

- Account for **Structural Modeling Uncertainties** (outside scope of paper)
- Evaluate **Seismic Collapse Performance** *(Section 7)*

3 DEFINE GOALS OF COLLAPSE ANALYSES AND DEFINE COLLAPSE METRICS

The first step in any structural performance assessment is to clearly identify the limit states of interest and how these will be evaluated. For collapse assessment, the goal is to establish a building's level of safety against earthquake-induced collapse, which may occur due to sideway collapse or loss in vertical load carrying capacity. In either case, the collapse risk is typically quantified by first calculating a collapse fragility curve, which relates the likelihood of collapse to the input ground motion intensity (discussed later in Section 6.4). The collapse fragility curve can then be used to assess one or more of the following metrics:

- Median collapse capacity, or "collapse margin" which is the ratio of the median collapse capacity to some ground motion demand level of interest. In the United States, the demand level of interest for assessing collapse is commonly the Maximum Considered Earthquake (MCE) ground motion.
- Probability of collapse for a specified demand level of interest (e.g. the probability of collapse at the MCE ground motion intensity).
- Mean annual rate of collapse, which involves integrating the collapse capacity distribution with the ground motion hazard curve.

4 CREATE STRUCTURAL MODEL CAPABLE OF SIMULATING COLLAPSE

When creating a structural model for simulating collapse, the focus is notably different as compared to creating a structural model for other purposes (e.g. for serviceability analyses under small deflections up to the onset of structural yielding). At the component level, parameters that are very important for predicting serviceability limit states, such as member yield strength and initial stiffness, may be less important for predicting collapse; whereas other parameters, such as component deformation capacity, cyclic deterioration and energy dissipation capacity may be critical for accurate simulation of collapse. At the structural system level, collapse simulation models must be capable of capturing large deformations and localized degradation effects that contribute to the development of structural collapse mechanisms.

4.1 Understand Damage and Failure Modes

The first step in creating a robust structural model is to understand all possible deterioration and failure modes of the structural system, and then to account for these modes when creating the structural model. This includes possible collapse modes of the primary lateral force resisting system (e.g. flexural hinging or shear failure in beams and columns), as well as possible collapse modes of the gravity system (e.g. punching shear failure in the concrete slab, and the vertical collapse that may follow).

To illustrate this thought process, the following is a summary of deterioration and collapse modes in RC frame structures (following Liel and Deierlein 2008):

- Component deterioration leading to a sidesway collapse mechanism:

- Column and beam flexure hinging or column shear failure
- Joint shear failure
- Failure of reinforcement anchorage or lap splice
- Or a combination the above mechanisms.

- Component deterioration leading to loss of vertical load carrying capacity:
  - Loss of gravity-load bearing capacity in a column after it has failed in shear
  - Punching shear and vertical collapse at slab to column connection
  - Column crushing and axial failure due to combined gravity load and overturning effects.

The extent to which these failure modes are possible and/or likely in a specific building will depend on the extent to which capacity design principles and ductile seismic detailing were employed in the original building design.

4.2 Structural System Model for Predicting Collapse

The simulation model must be carefully created to account for all of the possible collapse mechanisms in a particular structural system. Key considerations for component modeling are described below. At the system level, an important choice is whether the model is two or three-dimensional. Typically, most building specific studies will warrant three-dimensional models, since the structural configurations invariably have features that will create three-dimensional deformations and nonlinear redistribution of forces between the various structural components. On the other hand, more generic studies of archetypical structures, which may be undertaken to evaluate certain building code provisions, can often be conducted with two-dimensional models.

Figure 1 shows an example of a model that could be used for a three-bay frame building. This model includes nonlinear beam and column elements, nonlinear joint elements with finite size, and a leaning column to account for the destabilizing P-Δ effects resulting from gravity loads which are not directly applied to the frame (e.g. loads on the gravity system).

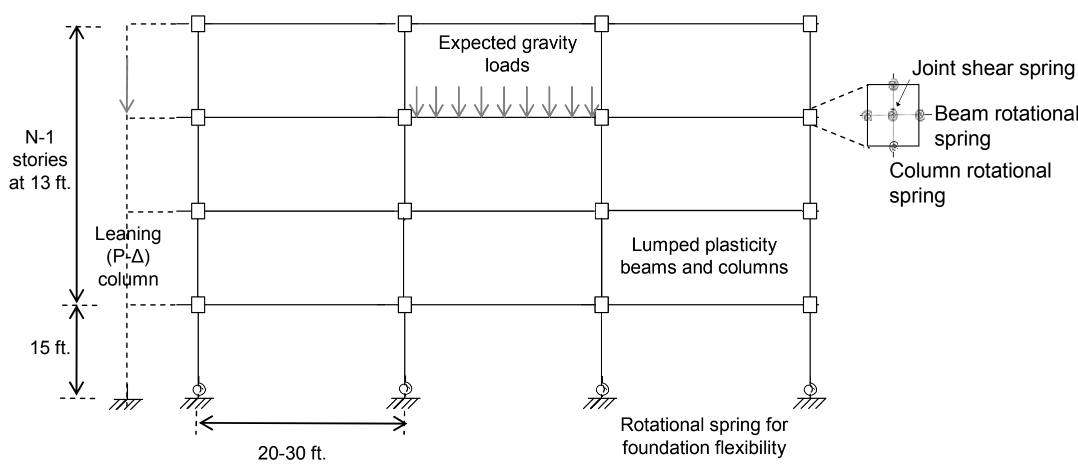


Figure 1: Example of overall structural system from for a frame building (after Liel and Deierlein 2008).

4.3 Decide which Models to Utilize for Each Component of System

Once the overall model of the structural system is outlined, then decision need to be made regarding appropriate models for each component of the structural system. For example, in the above frame (representing perhaps a non-ductile or modern RC frame buildings), models must be created for beam, columns, and joint components. When determining the appropriate component models, one must focus on the *specific purpose* of the structural model. For example, modeling decisions will be entirely different if the purpose of the model is to (a) directly simulate structural collapse (where extreme levels of deterioration are important), or to (b) predict displacement response at low levels of ground motion (where concrete cracking and tension stiffening would be important).

For example, consider a RC frame structure with flexural-dominated beam columns and strong joints, such as a well-detailed modern special moment resisting frame system. For this building, there are two primary alternatives for modeling beam-column response: fiber models (as shown in Figure 2) and lumped plasticity elements (as shown in Figure 3). The fiber elements are based on detailed stress-strain models for each material (concrete and steel) and can capture cracking, the onset of yielding, and then the spread of plasticity throughout the elements cross-section and along the element length. The lumped plasticity models are simpler and represent the plasticity using a nonlinear moment-rotation spring at each end of the element. Between the two ends, the member is assigned quasi-elastic properties with effective stiffness parameters that account for distributed cracking and bond slip response.

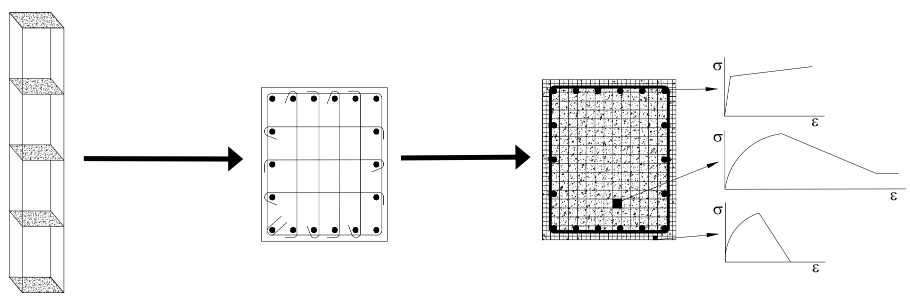


Figure 2: Overview of the fiber-element structural model for a modern RC frame building (after Haselton et al. [2008a]).

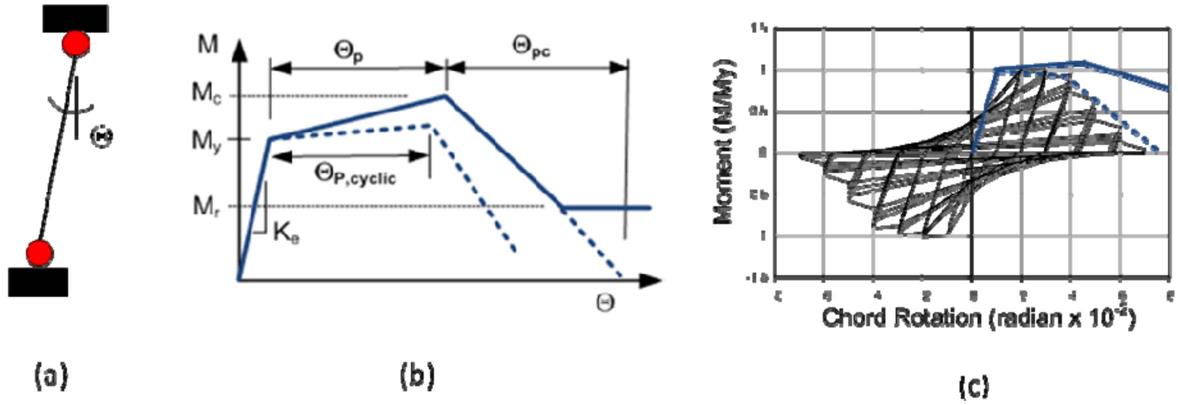


Figure 3: Overview of lumped-plasticity element structural model (a) model idealization, (b) generalized moment-rotation curve, and (c) cyclic hysteretic response (after Haselton et al. [2008a]).

Based on initial impressions, the detail and perceived accuracy of the fiber model suggests that it should be the preferred choice for nonlinear analysis, however, for the reasons outlined below, the lumped-plasticity model does a better job at simulating strength and stiffness degradation associated with *structural collapse*. In flexurally-dominated RC frame structures, collapse is caused by strain-softening in beam-column elements due to the combined effects of concrete crushing, confinement tie yielding, and longitudinal reinforcing bar yielding and buckling [Haselton et al. 2008b]. Even though fiber models can capture basic deterioration of the concrete, research has not progressed to the point of providing fully calibrated and validated models to capture strain softening associated with buckling and fracture of the longitudinal steel reinforcing bars. Recent research by Brown, Lehman, and Stanton [2007] has examined the factors that influence reinforcing bar buckling, and several researchers have explored ways to model buckling behavior in fiber type analyses [Kunnath et al., 2009, Gomes and Appleton, 1997]. However, to date these models are not fully developed to the point of accurately simulating the full extent of strength and stiffness degradation at large hinge rotations. Since such buckling is a major contributor to flexural strength degradation (particularly in lightly loaded columns with large steel areas) existing fiber-type models are not sufficient for use in direct collapse simulation. In contrast, if the analytical objective was to predict the onset of cracking and yielding at service-level ground motions, then the fiber would be the model of choice.

In comparison to fiber-type models, the lumped plasticity model loses some of the precision of the physical interpretation of concrete cracking and yield initiation, but it can capture strain softening, as shown by the descending slope on the force-displacement curve in Figure 3. This component model can be calibrated to capture the effects of reinforcing bar buckling and, therefore, is well suited for direct collapse simulation.

4.4 Modeling Components with Non-Simulated Collapse Modes

While, ideally, all significant failure models should be directly simulated in the analytical model, as a practical matter it is not always feasible to do so. In some cases there models are limited by basic lack of knowledge to accurate simulate certain phenomena, and in other cases, the limitations have more to do with practical limitations of available simulation software. For example, research by Elwood and Moehle [2003] has provided basic information and models to quantify the loss of gravity load bearing capacity in nonductile concrete columns after they have failed in shear, but accurate simulation of this behavior in the context of large indeterminate frame systems remains a challenge. In the case that not all modes are directly simulated, the analyst should account for the "non-simulated modes" through post-processing checks to avoid non-conservative prediction of collapse (associated with the exclusion of a particular failure mode). In the case of non-simulated collapse modes, a component limit state check can be used to conservatively identify whether or not this collapse mode occurred in dynamic analysis. Continuing the example above, several researchers (Elwood and Moehle, 2003; Aslani and Miranda, 2003) have shown that gravity collapse of nonductile concrete columns is associated with the shear and deformation demands on a particular column. Experimental data can be used to determine when this gravity collapse may reasonably be expected to occur (depending on the properties of a particular column).

Owing to the limited capabilities of existing simulation models, many structural systems will generally require checks for non-simulated collapse modes that are not modeled directly in the analysis. However, the post-processing approach to non-simulated collapse modes notably neglects the effect of redundancy and force redistribution leading to the progressive collapse of the structure, and hence, these checks tend to be conservative. In the future, it is anticipated that there will be less reliance on non-simulated collapse checks as the modeling capabilities and robustness of nonlinear analysis methods improve.

4.5 Calibrate Component Models with Focus on Predicting Collapse

Calibration Goal

Each component of the structural model must now be calibrated to ensure that the predicted force-displacement response of the component represents that observed for such a component in laboratory tests. The goal of this calibration process is to create a structural model that accurately captures the strength and stiffness deterioration of the structural component based on its specific design details (i.e. RC columns with more or less transverse reinforcement). Since collapse is driven by strength deterioration and strain softening, the calibration process should focus on these aspects of component response and ensure that they accurately match observations from test data that will contribute significantly to structural collapse.

Example Calibration

For the example of an RC beam-column element, Figure 4 shows an example of cyclic test data and model calibration. This figure shows the experimentally observed response, the analytically calibrated response, and for comparison also shows the response of the calibrated model if subjected to a monotonic load (black bold solid line). This example comes from a report by the authors that focused on calibration of 255 RC column elements with different design and detailing characteristics [Haselton et al. 2008b].

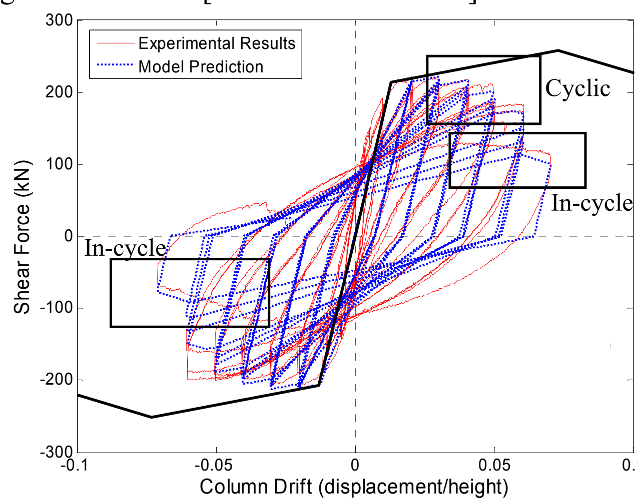


Figure 4: Example of calibration procedure; calibration of RC beam-column model to experimental test by Saatcioglu and Grira (1999), specimen BG-6 (after Haselton et al. [2008b]).

Two Fundamentally Different Types of Strength Deterioration to Keep Separate

Careful inspection of the cyclic response shown in Figure 4 reveals that there are two type of strength deterioration, as shown by the labeled regions in the figure. Stiffness degradation is also evident, but is less important for collapse response and is not discussed here. The two types of strength deterioration are explained in several references [Ibarra et al. 2005, 2003, etc.], but the simplest explanation, as given in Chapter 4 of FEMA 440 [2005], describes the two types of deterioration as follows:

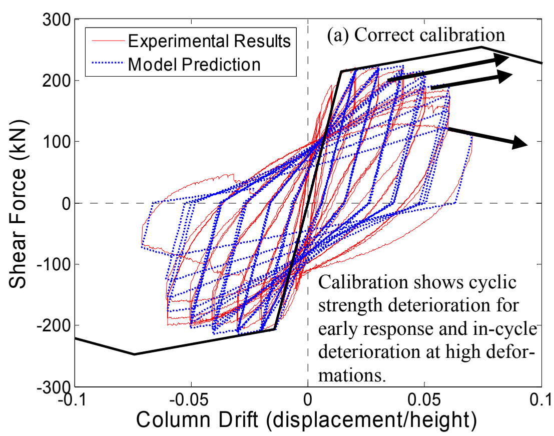
- *In-cycle strength deterioration*: In this mode, strength is lost in a single cycle, which means that the element exhibits a *negative stiffness*. This type of strength deterioration is critical for modeling structural collapse [Ibarra et al. 2005, 2003].
- *Cyclic strength deterioration*: In this mode, strength is lost between two subsequent cycles of deformation, but the *tangent stiffness remains positive.* This type of strength deterioration is less important (relative to in-cycle) for modeling structural collapse [Ibarra et al. 2005, 2003; Chapter 5 of Haselton and Deierlein 2007].

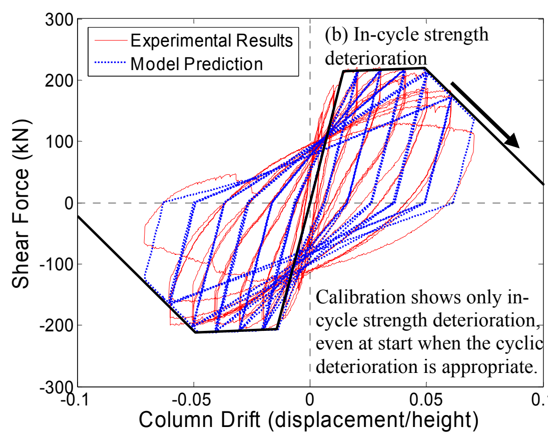
As labeled in Figure 4, *cyclic strength deterioration* is observed in the cycles before 5% drift and *in-cycle strength deterioration* in the two cycles that exceed 5-6% drift. The component model must be calibrated so that it can properly predict when both type of strength deterioration will occur for a particular element in the structure.

Avoiding the Common Calibration Pitfalls

Lack of distinction between these two fundamentally different modes of deterioration can lead to large errors in the resulting collapse predictions. To investigate the significance of improperly modeling the strength deterioration, we calibrated an analysis model to the above test data in two incorrect ways and then used the resulting models to conduct collapse predictions for single degree-of-freedom systems.

Figure 5 shows the specimen calibrated in three ways. Figure 5a replicates Figure 4, and shows the correctly calibrated component model. The arrows indicate how the component model would response to a monotonic deformation during each part of the cyclic response. Figure 5b shows the specimen calibrated incorrectly with all strength loss caused by in-cycle strength deterioration. Notice that this method of calibration causes the negative failure slope to be reached at a lower drift level and leads to a steeper post-failure slope than in Figure 5a. Figure 5c, the same specimen is calibrated incorrectly with all strength loss caused by cyclic strength deterioration. In this case, the element never experiences a negative stiffness; therefore, when calibrated this way, dynamic instability can occur only with a combination of P-delta and severe cyclic strength loss.





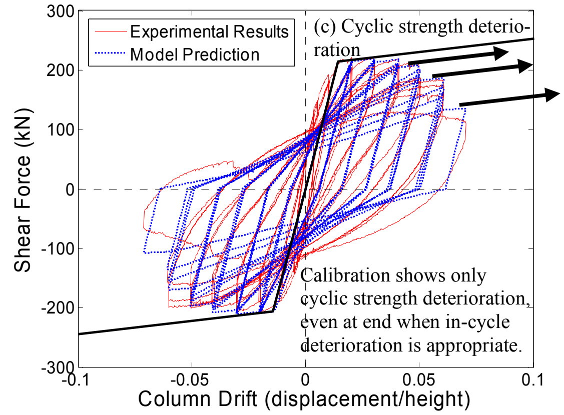


Figure 5: Illustration of a) correct calibration, b) incorrect calibration using only in-cycle strength deterioration, and c) incorrect calibration using only cyclic strength deterioration (Saatciolgu and Grira [1999] specimen BG-6) (after Haselton et al. [2008b]).

Using the three calibrations from Figure 5, we created three single degree-of-freedom (SDOF) models, each with an initial period of 1.0 sec, a yield spectral acceleration (at 1 sec) of 0.25g, a damping ratio of 5%, and an axial load resulting in a relatively low amount of P-delta (stability coefficient of 0.02). The results of the time history of drift response for one ground motion, scaled to two different intensity levels, are shown in Figure 6, illustrating the importance of carefully calibrating deterioration. At 1.0g, the model calibrated with Method B (all in-cycle deterioration) collapses prematurely, whereas the other two models do not collapse and display similar drift responses. At 2.6g, the models calibrated with Methods A and B (correct, all in-cycle) collapse, while the model calibrated with Method C (all cyclic) does not collapse because it is calibrated without the negative post-failure stiffness (a nonconservative response). If these three single-degree of freedom structures are subjected to incremental dynamic analysis using a set of 20 ground motions [Haselton and Baker 2006], Method B leads to a 65% under-estimation of the median collapse capacity, relative to the more correct Method A. Method C over-estimates the collapse capacity by 97%, compared to Method A (See Haselton et al. 2008b for more details). Note that the differences in calibration become evident only when the earthquake ground motion intensities are increased up to the point of collapse. At lesser intensities, the three models give comparable results.

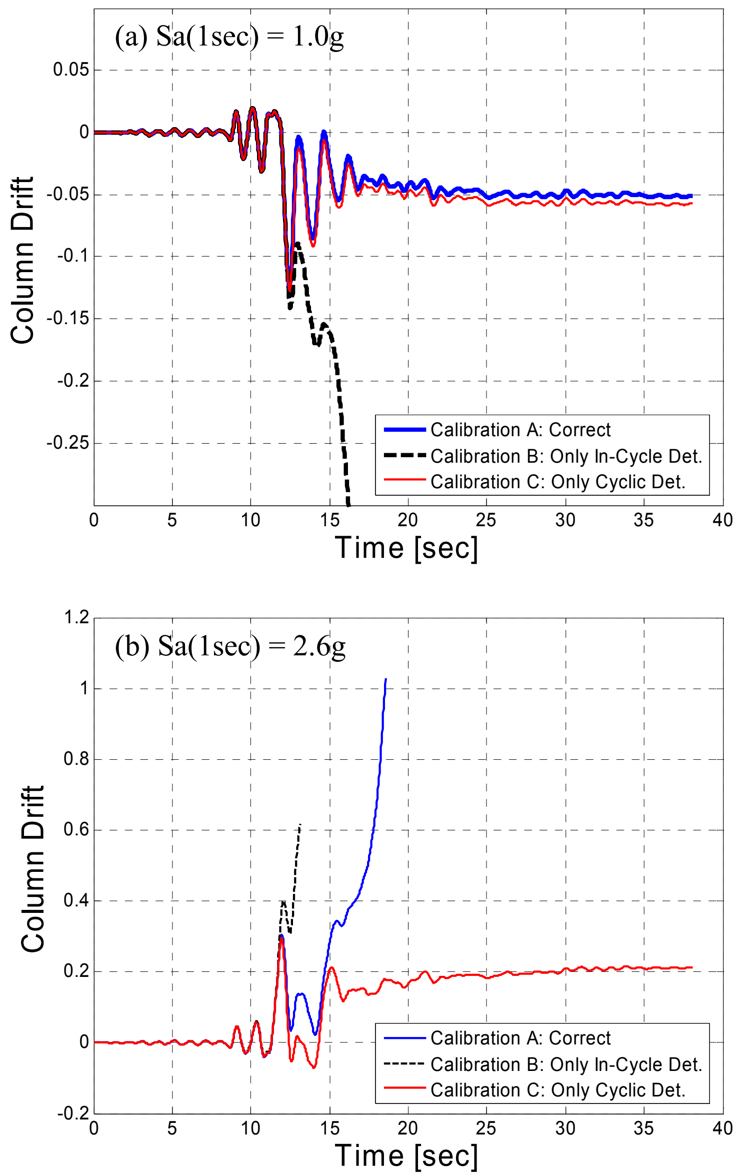


Figure 6: Time-history drift responses for three SDOF systems calibrated in Figure 5 showing drift response for Sa(1sec) = 1.0g and (b) showing response for Sa(1sec) = 2.6g (after Haselton et al. [2008b]).

Optimal Set of Data for Calibration

As mentioned above, most of the data available in the literature is from tests with symmetrical cyclic loading. This presents challenges for calibration since (a) the separation of in-cycle versus cyclic deterioration is less clear, and (b) it is impossible to understand the effects that a varied loading protocol may have on component response. The recent FEMA P440 [FEMA 2005] document discusses this issue and suggests that it is ideal to have test data were multiple loading protocols are used for identical test specimens, since this would provide important information regarding the effects of the loading protocol. To expand on this, the following set of test data (all for identical test specimens) would be an example of an ideal set for purposes of calibration for collapse simulation. Each of these tests would ideally continue to large deformation levels, so the near-collapse response behavior of the element can be quantified.

- Monotonic protocol (to calibrate the response of the component with only in-cycle deterioration). In the example shown in Figure 4, only cyclic response data were available, so the monotonic response of the calibrated component model is based on extrapolation from the cyclic data. This approach is approximate at best, but is often necessary because most available data is cyclic.
- Various symmetric cyclic loading protocols (to calibrate the cyclic deterioration behavior):
  - One cycle per displacement level.
  - Three cycles per displacement level.
  - A protocol with many cycles (to assess the possible significance of fracture/fatigue problems that may not be present in the other protocols).
- Several cyclic protocols with a few cycles followed by a monotonic push (to ensure that in-cycle versus cyclic deterioration are properly separated in the calibration). Various unsymmetrical cyclic loading protocols (to ensure that the calibrated component model is robust and properly handles these cases).

5 CLEARLY DEFINE COLLAPSE LIMT STATE

When referring to the prediction of seismic *collapse*, it is important to qualify the definition of collapse, since different documents and authors may have different interpretations of collapse. For example, in assessment of existing buildings according to ASCE/SEI 41 [2006], *collapse* can be defined to occur when a single component exceeds a particular threshold, often identified by a plastic rotation demand. In many cases, these demands structure would not actually cause the structure to become structurally unstable, but instead imply a conservative check of the so-called *near collapse* or *collapse prevention* limit state.

In this paper, *collapse* is defined as the point that the structure becomes unstable and is either at or beyond the collapse threshold. Specifically, the collapse state is identified either (a) by the ground motion intensity at which point the story drifts increase without bounds for a very small increase in ground motion intensity, or (b) by a non-simulated collapse mode that is intended to represent the onset of a vertical collapse that is triggered by large story drifts combined with gravity loading (e.g. a slab of a gravity frame disconnecting from the column).]

To create specific and measurable definitions of collapse, a simulated collapse can be identified by a drift limit at which it is not possible for the building to still be stable, and this can be generally identified through use of pushover analysis and/or looking at the results of dynamic analysis results. Figure 7 shows the results of static pushover analysis for the example structure. This shows that zero base shear capacity would be reached at between 8-10% roof drift and something near a 15% maximum story drift ratio. Accordingly, the quantitative collapse limit state could be defined as the building reaching 20% story drift, i.e., well beyond the point where the structure looses sidesway stability. A non-simulated collapse would be triggered by the exceedance of some deformation demand in a portion of the building (e.g. drift in a gravity frame, shear distortion of a joint, etc.). An important point is that the simulated collapse limit state is defined explicitly and is not simply defined as the point at which the *numerical simulation fails to converge*. Whereas non-convergence has frequently been cited to define *collapse* in past research, it can potentially lead to very conservative estimates of collapse capacity that depend on the specific solution algorithm used. To predict *collapse*, the simulation must typically fully converge at large enough displacements to represent the collapse limit state (e.g. story drift ratios on the order of 10% to 20% for ductile frame systems). If convergence is not achieved to this level of deformation, it is difficult to distinguish between non-convergence due to limitations of the solution algorithm or non-convergence resulting from dynamic instability as the structure collapses. Achieving numerical convergence to large deformation levels is often very difficult due to highly nonlinear behavior and negative stiffness in some components, but assuming that the analysis model is well formulated (with robust force-recovery algorithms), it is possible to develop robust solution methods. Solution strategy algorithms employed by the authors are outlined in a later section.

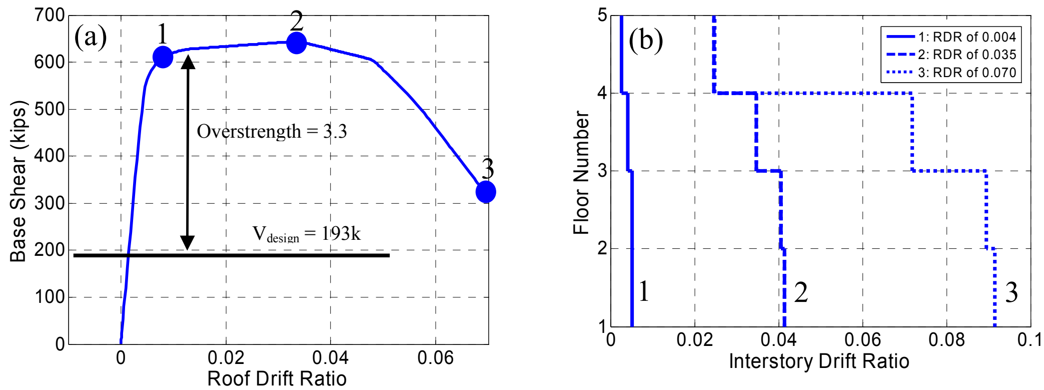


Figure 7: (a) Monotonic static pushover, and (b) peak interstory drift ratios at three deformation levels during pushover (after Haselton and Deierlein [2007]).

6 COMPLETE NONLINEAR DYNAMIC COLLAPSE ANALYSES

Once the structural model has been developed and verified, and the collapse criteria have been established, the structural model can be used to predict the distribution of collapse capacity for the building. In addition to a robust structural model, this necessitates a set of ground motions and a robust numerical solution algorithm, both of which are discussed in this section.

6.1 Ground Motion Selection and Scaling for Collapse Analyses

For collapse assessment, selection or synthesis of ground motions to obtain time histories representative of ground motions that may cause structural collapse is critical. Of particular importance are choices between recorded and synthetic ground motions and scaling of records to obtain ground motions strong enough to cause structural collapse. Given uncertainty about frequency content, duration and other characteristics of future ground motions, a fairly large set of ground motions must be used to predict collapse (approximately 40 records is typically judged to be sufficient for estimation of mean and dispersion in collapse capacity). For more information, readers are referred to a related paper by the authors [Haselton, et al. 2009], focusing on selection and scaling of ground motions for collapse simulation, including the unique spectral shape characteristics of high intensity ground motions with long (infrequent) return periods.

6.2 Develop Robust Solution Algorithm

Numerical Solution Algorithm

As mentioned previously, the numerical solution algorithm is a challenging aspect of the collapse simulation. Prediction of collapse requires solution of system of equations at each time step in dynamic analysis, which becomes problematic when the structure is highly degraded such that the stiffness matrix becomes ill-conditioned by elements that have small or even negative stiffnesses. If the algorithm is not robust, the nonlinear solution algorithm will interrupt the analysis before the collapse limit state has been reached.

The following is pseudo-code for the algorithm that the authors have developed for direct collapse simulation. This code applies to a single time-step of the time-history analysis. The authors specifically utilize the OpenSees [2009] structural modeling platform, but this concept can also be carried to other platforms, with proper modifications to utilize the specific algorithms of that platform. The general approach involves specifying a tolerance related to the norm displacement increment [OpenSees 2009]. A variety of nonlinear solution algorithms (e.g. Newton-Raphson) may be used to satisfy the dynamic equation of equilibrium at each step. If the solution algorithm chosen cannot solve the system of equations to the desired accuracy (as specified by the tolerance) within a reasonable number of iterations, different solution algorithms are attempted; the order of algorithms shown below is just based on the experience of the authors, and could be further refined. If a solution can still not be obtained, the specified tolerance (allowable error) is increased. Additionally, to avoid wasting computing time trying to solve an unsolvable system of equations, singularity is checked at each step of the analysis, and the analysis is stopped if singularity is detected. If convergence is obtained, then the final tolerance at the converged state is recorded for later reference as needed when interpreting the simulation results. Further descriptions about the solution algorithms used in the pseudo code described below are available in OpenSees documentation [2009] and standard dynamic analysis references.

```
Select starting tolerance: 1.0 * 10^-7
Select maximum allowable tolerance: 1.0 * 10^-1

Current tolerance = starting tolerance
While (Current tolerance ≥ maximum allowable tolerance) AND (the solution is non-converged)
        Try Newton Line Search Solution Algorithm to obtain converged solution at the specified tolerance;
        If (still non-converged)
                Try Newton Solution Algorithm; Sometimes different solution algorithms are able to
                achieve convergence
        If (still non-converged)
                Try Modified Newton Solution Algorithms;
        If (still non-converged)
                Try Krylov Newton Solution Algorithm;
        If (still non-converged) reduce allowable tolerance,
        Check for a singularity, if a singularity has occurred, report this and stop analysis.
end

If (still non-converged)
        Stop analysis and report non-convergence
else
        Save the final convergence tolerance to file (for later checks as needed)
        Check for collapse, and if collapse has occurred stop analysis and report collapse.
        If not collapsed, then move to next time-step of analysis
end
```

Note that prediction of collapse can be very time-consuming even on a relatively fast computer, given the large number of iterations and possibility of having trying several different convergence algorithms.

Even with a robust numerical solution algorithm, problems may still occur when attempting to solve the system of equations when the building is in a highly degraded state. In the case that non-convergence or singularity occurs before the building reaches the collapse deformation level, the authors often slightly perturb the scale factor on the earthquake ground motion (slightly perturb the spectral acceleration level). Such slight perturbations can often lead to a converged solution by getting around the numerical issues that have randomly occurred at a given level of spectral acceleration.

Avoiding Numerical Problems Resulting from a Sparse Mass Matrix

When creating a model with many degrees of freedom (such as a frame model with joint elements, beam elements, column elements, etc.) the stability of the numerical solution is affected by how the building mass is allocated to the degrees of freedom. If the floor mass (e.g. slab mass) is only distributed between a few of the degrees-of-freedom as each floor level, then the mass matrix becomes sparse, and the generalized stiffness matrix tends to become ill-conditioned more regularly when attempting to complete the nonlinear time history analyses. We have found that adding small terms to the mass matrix solves this problem. To this end, we spread at least small portions of the mass to all translational and rotational degrees-of-freedom, and this substantially improves the numerical stability of the solution.

Figure 8 illustrates distribution of masses to many degrees of freedom for an RC frame structure. The majority of the mass is placed at the top of the joint (since this is closest to where the mass of the slab acts), and then small masses are placed at all of the other translational and rotational degrees-of-freedom. For these small translational masses, experience has shown that the 0.005% of the primary mass (i.e. the mass placed at the top of the joint) is sufficient. For the rotational components of mass, this same 0.005% mass can be used, with the assumption that the mass is spread over the length of the joint region (in order to compute the rotational mass quantity). The above suggested values are not unique, and are just suggestion based on the author's experience, in order to avoid numerical problems related to a sparse mass matrix.

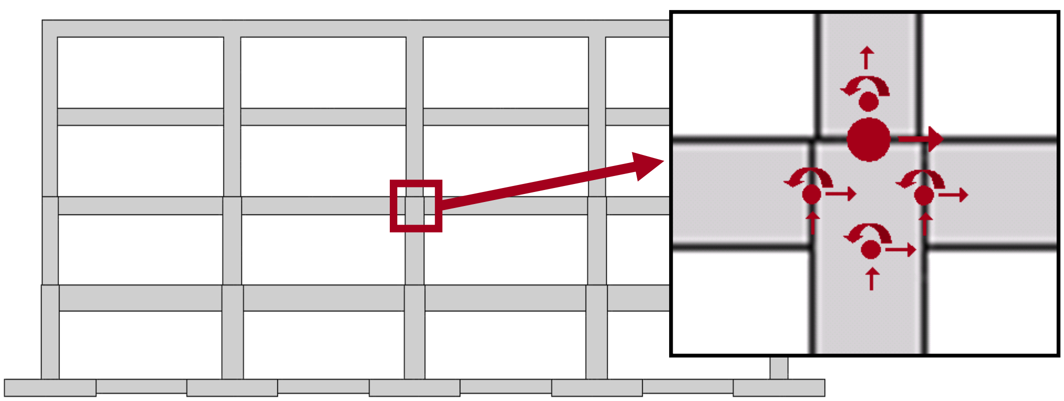


Figure 8: Illustration of small masses placed at all degrees-of-freedom in the joint model of a frame building, for the purpose of avoiding numerical instabilities caused by a sparse mass matrix.

6.3 Damping Considerations for Nonlinear Analysis

The approach to modeling damping is also important for nonlinear analyses. Recent studies have shown that some common approaches can lead to unrealistic spurious damping forces and/or inappropriate levels of effective damping once the system has become nonlinear. This topic is addressed in detail in the recent ATC-72-1 report [ATC 2008]; for brevity, the reader is referred to the ATC-72-1 report, and the modeling guidelines for damping are not reiterated here.

6.4 Complete Incremental Dynamic Analyses to Determine Distribution of Collapse Capacity

Once the numerical issues are addressed, then Incremental Dynamic Analyses (IDA) [Vamvatsikos and Cornell 2002] can be used to determine the distribution of collapse capacity. Figure 9 illustrates the results of IDA for a 4-story building with 0.86 second fundamental period (building ID1010 from Haselton and Deierlein [2007]), showing that IDA produces the relationship between ground motion intensity (e.g. Sa) and structural response (e.g. maximum story drift).

In order to determine the collapse capacity for each earthquake record, we take the lesser of the spectral acceleration at the specified collapse drift (e.g. 20% interstory drift) and the spectral acceleration associated with a non-simulated collapse mode, as discussed in section 4.2. For illustration purposes, Figure 10 shows the distribution of collapse capacity that would result if non-simulated collapse modes were not present, or if simulated sidesway collapse always governed. This cumulative distribution function (CDF) shows a median collapse capacity of Sa(0.86s) = 2.24g and a logarithmic standard deviation of $\sigma_{LNSa(0.86s)} = 0.42$. At the example site used for this structural design, the Maximum Considered Earthquake (MCE) intensity is Sa(0.86s) = 1.05g. This results in a margin against collapse (at the MCE) of 2.24g/1.05g = 2.13, and a conditional probability of collapse at the MCE of 9%. The mean annual frequency of collapse can also be computed, by integrating the collapse CDF with the hazard curve, and this is 2.5×10^{-4} collapses/year for this example site.

To complete this IDA, typically between 10-20 nonlinear dynamic analyses are completed for each ground motion record (at the various levels of spectral acceleration). Using a 2009-era desktop with a dual-core processor, these analyses for the 44 ground motion records used here takes 0.5 to 1.0 days for a four-story two-dimensional frame, and this time increases to approximately three days for a 20-story frame.

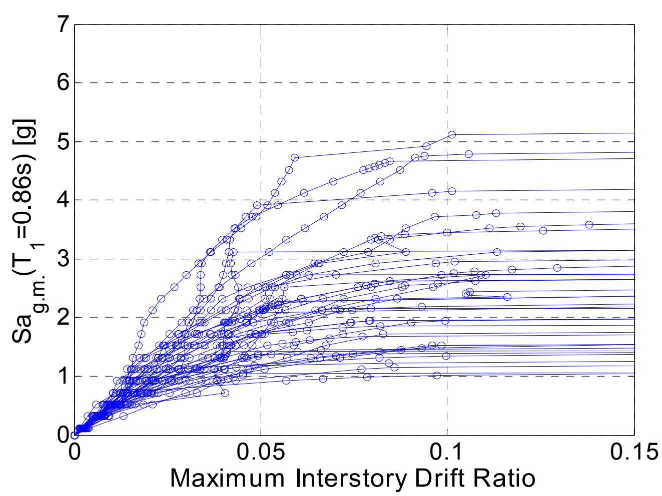


Figure 9: Results of Incremental Dynamic Analysis for a 4-story modern RC frame building (after Haselton and Deierlein [2007]).

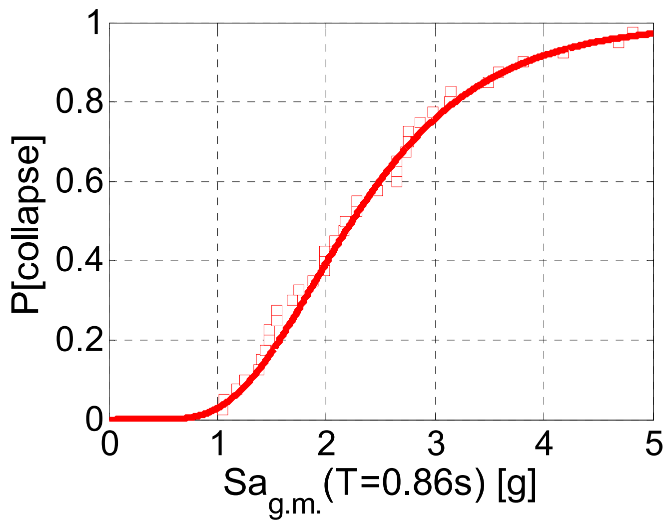


Figure 10: Collapse capacity distribution (modified from Haselton and Deierlein [2007]).

6.5 Verification of Structural Response Results

As with any structural analysis, it is important to look closely at the structural response results to ensure that there are no obvious errors that are masked by the summary statistics. For example, the issue of the numerical problem associated with the sparse mass matrix (section 6.2) can not be seen in the collapse capacity summary statistics, because when the numerical problem occurs, the solution reports a large displacement response (which is judged to be collapse) even though that large displacement response is unrealistic. This problem was uncovered by looking at the structural responses and noticing that the building response was stable at a given level of spectral acceleration (which had modest element plastic rotation demands), and then at a slightly higher level of spectral acceleration the building collapsed. This behavior was unrealistic and indicated the problem with the numerical solution

For a selected ground motion record, Figure 11 shows an example of this showing the interstory drifts and locations of plastic hinging for another 4-story frame building (with 1.0 second period; from Haselton et al. [2008a]). Figure 12 shows the plastic hinge response of a typical column, located in the third story. Each of these checks show that, as the ground motion level increases, the structural model is behaving as expected, and this provides confidence in the validity of the structural collapse predictions.

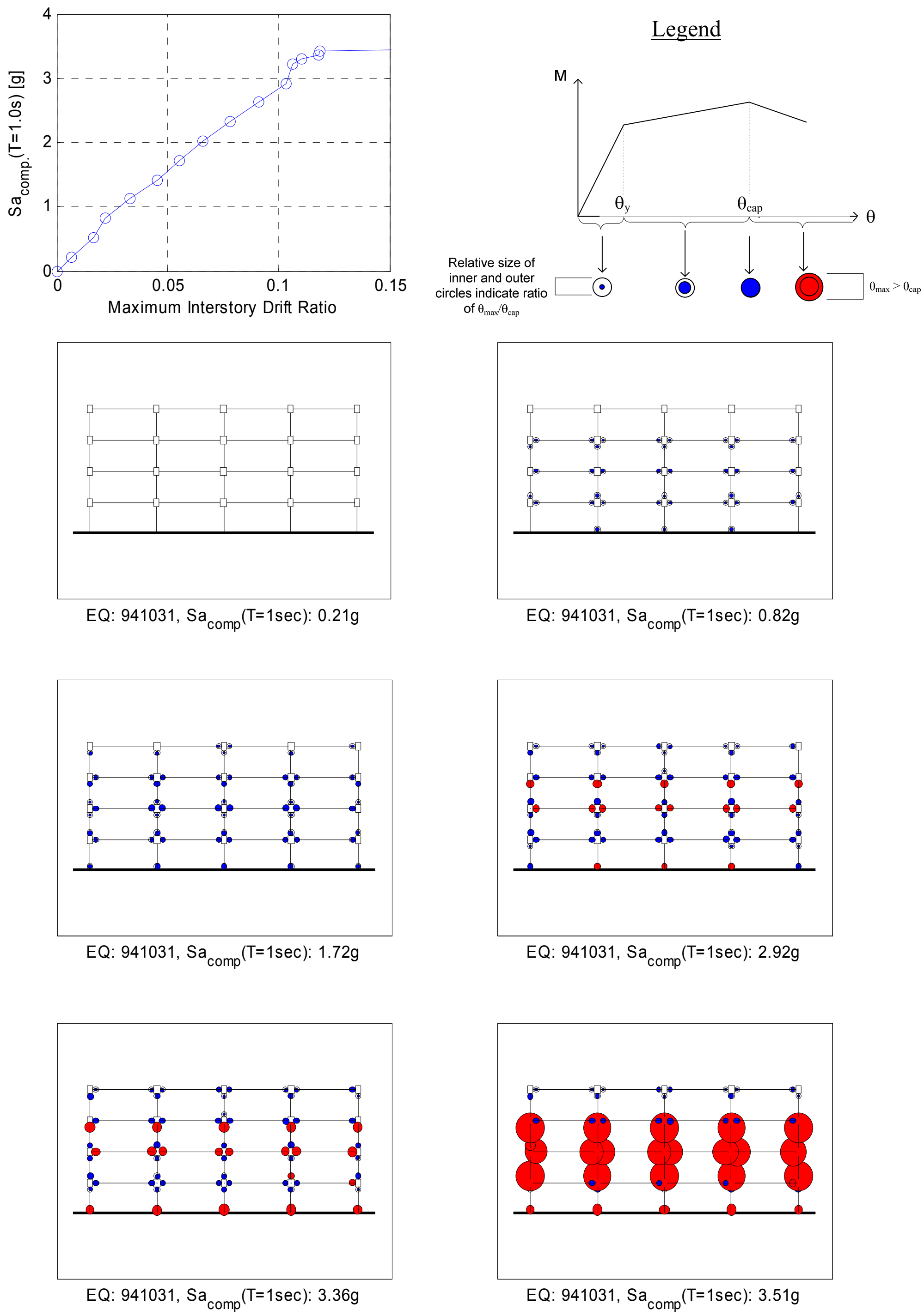


Figure 11: Diagrams showing progression of peak interstory drift and damage from low levels of ground motion to collapse for a single selected earthquake record . Record causes damage to localize in second and third stories, and causing collapse at S_a(T=1 sec) = 3.5g (from Haselton et al. [2008a]).

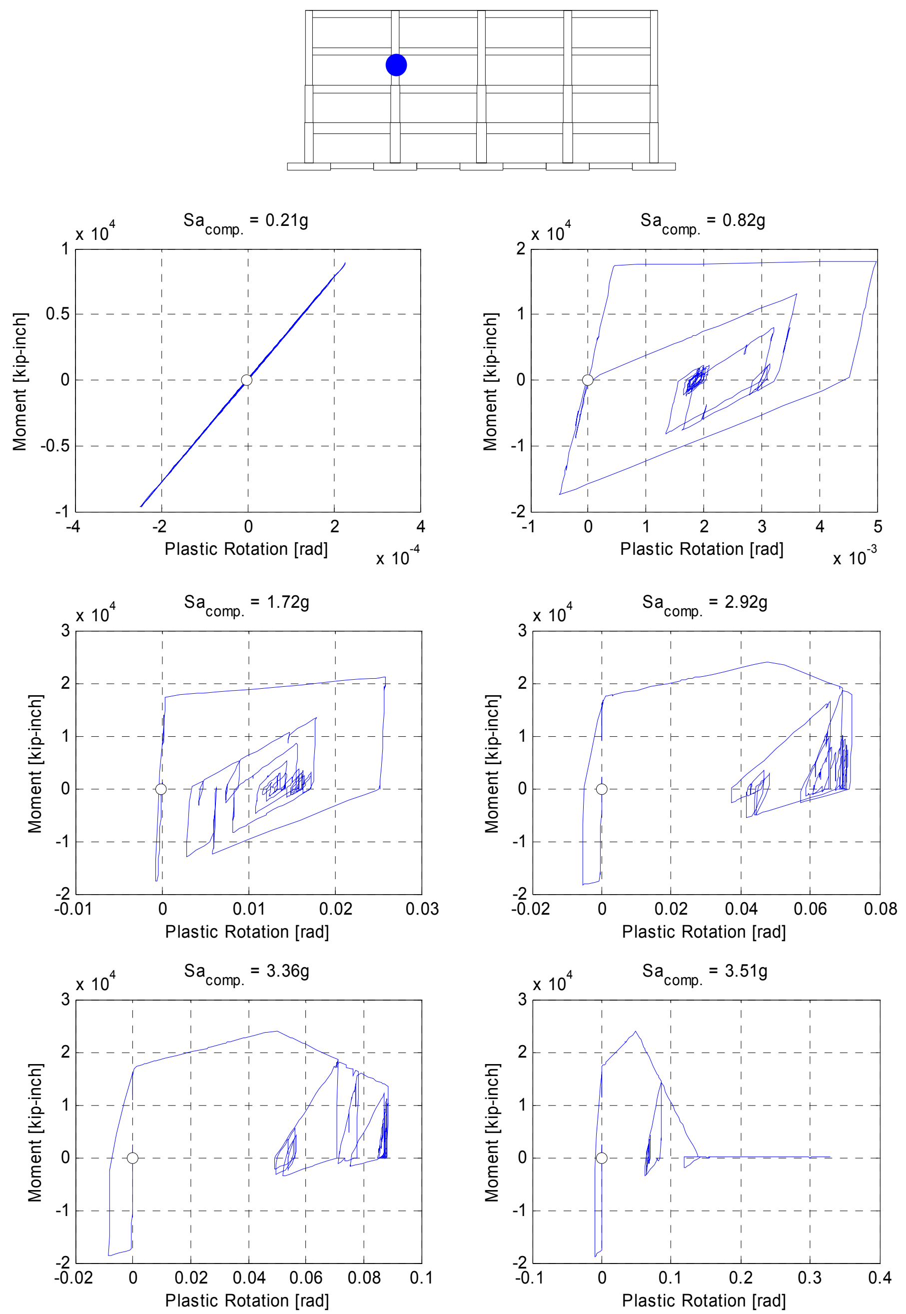


Figure 12: Diagrams showing column responses for various levels of ground motion (from Haselton et al. [2008a]), for the same earthquake record as above. Similar figures can be obtained for response of beams, joints, or other components in the structure.

7 EVALUATE COLLAPSE PERFORMANCE

In order to evaluate the collapse performance, one must compute the collapse metrics of interest, and then compare these collapse metrics to some type of standard of safety. As outlined in Section 3, the following three collapse metrics are commonly utilized in collapse performance assessments.

- Median collapse capacity, or collapse margin.
  - For the example in section 6.4, the collapse margin is 2.13 for the MCE demand.
- Probability of collapse for a demand level of interest (e.g. the MCE demand).
  - For the example in section 6.4, this is 9% for the MCE demand.
- Mean annual rate of collapse, which involves integrating the collapse capacity distribution with the ground motion hazard curve.
  - For the example in section 6.4, this is $2.5x10^{-4}$ collapses/year.

Even with these collapse capacity metrics computed from a structure, it is difficult to determine if the structure is "safe enough" because there are no standard criteria for the collapse safety of structures. It was not until recently that collapse performance criteria been proposed by the FEMA P695 project [FEMA 2009] for new structural design. The FEMA P695 project has proposed that a conditional collapse probability of 10% (conditioned on the MCE demand) is an acceptable level of collapse safety for new construction. Notice however, that these target collapse probability goals incorporate other uncertainties that are not discussed in this paper (e.g. structural modeling uncertainties, etc.), but which are important for consideration in collapse performance assessment. These uncertainties are discussed in the FEMA P695 document, but are discussed in more detail in recent work by the authors [Liel et al. 2009].

8 SUMMARY AND REMAINING RESEARCH NEEDS

This paper has presented a summary of the important issues involved in simulating structural collapse, and provided some guidance and suggestions on best practices, as based on the recent collapse assessment research by the authors.

From this discussion, the following topics are some that warrant future research in this research area. Note that the scope of this paper focused on structural *collapse simulation*, and did not address the many other important factors in a full *collapse performance assessment* (e.g. ground motion selection and scaling, treatment of uncertainties, etc.); accordingly this list of research needs focuses only on the topic of structural *collapse simulation*.

- Advance fiber-type modeling capabilities to allow models to capture the damage modes leading to structural collapse (rebar buckling, etc.).
- Improve the body of experimental data by completing testing programs that utilize many different loading protocols for identical specimens, and continue the testing to large levels of deformation so that the near-collapse response behavior of the components can be clearly observed.
- Improve component models (both improved models and more accurate calibration) based on the above improved experimental data.
- Work to evaluate the accuracy and limitations of collapse predictions with more complete comparisons to:

- System-level dynamic collapse experiments (will need to be developed to supplement recent work by Lignos et al. [2008]).
- Observed data from earthquakes (though such data is severely limited for modern buildings collapsing when subjected to large ground motions).

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